# ASSESSMENT DETAILS

## External assessment details 5 hrs 80%

## General

### Paper 1, paper 2 and paper 3

These papers are externally set and externally marked. Together they contribute 80% of the final mark for the course. These papers are designed to allow students to demonstrate what they know and what they can do.

## Calculators

### Paper 1

Students are not permitted access to any calculator. Questions will mainly involve analytic approaches to solutions, rather than requiring the use of a GDC. It is not intended to have complicated calculations, with the potential for careless errors. However, questions will include some arithmetical manipulations when they are essential to the development of the question.

### Papers 2 and 3

Students must have access to a GDC at all times. However, not all questions will necessarily require the use of the GDC. Regulations covering the types of GDC allowed are provided in the *Vade Mecum*.

## Mathematics HL information booklet

Each student must have access to a clean copy of the information booklet during the examination. One copy of this booklet is provided by the IBO as part of the examination papers mailing.

## Awarding of marks

Marks may be awarded for method, accuracy, answers and reasoning, including interpretation.

In paper 1, paper 2 and paper 3, full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations (in the form of, for example, diagrams, graphs or calculations). Where an answer is incorrect, some marks may be given for correct method, provided this is shown by written working. All students should therefore be advised to show their working.

## Paper 1 2 hrs 30%

This paper consists of section A, short-response questions, and section B, extended-response questions. Each section will be worth 15% of the total mark.

## Syllabus coverage

- Knowledge of **all** topics in the core of the syllabus is required for this paper. However, not all topics are necessarily assessed in every examination session.

 

## Mark allocation

- This paper is worth **120** marks, representing **30%** of the final mark.
- Questions of varying levels of difficulty and length are set. Therefore, individual questions may not necessarily be worth the same number of marks. The exact number of marks allocated to each question is indicated at the start of the question.

### Section A

This section consists of compulsory short-response questions based on the core of the syllabus. It is worth 60 marks, representing 15% of the final mark.

- The intention of this section is to test students' knowledge across the breadth of the core. However, it should not be assumed that the separate topics from the core are given equal emphasis.

### Question type

- A small number of steps is needed to solve each question.
- Questions may be presented in the form of words, symbols, diagrams or tables, or combinations of these.

### Section B

This section consists of compulsory extended-response questions based on the core of the syllabus. It is worth 60 marks, representing 15% of the final mark.

- Individual questions may require knowledge of more than one topic from the core.
- The intention of this section is to test students' knowledge of the core in depth. The range of syllabus topics tested in this paper may be narrower than that tested in section A.
- To provide appropriate syllabus coverage of each topic, some questions in this section are likely to contain two or more unconnected parts. Where this occurs, the unconnected parts will be clearly labelled as such.

## Question type

- Questions require extended responses involving sustained reasoning.
- Individual questions may develop a single theme or be divided into unconnected parts.
- Questions may be presented in the form of words, symbols, diagrams or tables, or combinations of these.
- Normally, each question reflects an incline of difficulty, from relatively easy tasks at the start of a question to relatively difficult tasks at the end of a question. The emphasis is on problem solving.

# Paper 2 2 hrs 30%

This paper consists of section A, short-response questions, and section B, extended-response questions. Each section will be worth 15% of the total mark.

## Syllabus coverage

- Knowledge of **all** topics in the core of the syllabus is required for this paper. However, not all topics are necessarily assessed in every examination session.

 

## Mark allocation

- This paper is worth **120** marks, representing **30%** of the final mark.
- Questions of varying levels of difficulty and length are set. Therefore, individual questions may not necessarily be worth the same number of marks. The exact number of marks allocated to each question is indicated at the start of the question.

### Section A

This section consists of compulsory short-response questions based on the core of the syllabus. It is worth 60 marks, representing 15% of the final mark.

- The intention of this section is to test students' knowledge across the breadth of the core. However, it should not be assumed that the separate topics from the core are given equal emphasis.

### Question type

- A small number of steps is needed to solve each question.
- Questions may be presented in the form of words, symbols, diagrams or tables, or combinations of these.

### Section B

This section consists of compulsory extended-response questions based on the core of the syllabus. It is worth 60 marks, representing 15% of the final mark.

- Individual questions may require knowledge of more than one topic from the core.
- The intention of this section is to test students' knowledge of the core in depth. The range of syllabus topics tested in this paper may be narrower than that tested in section A.
- To provide appropriate syllabus coverage of each topic, some questions in this section are likely to contain two or more unconnected parts. Where this occurs, the unconnected parts will be clearly labelled as such.

## Question type

- Questions require extended responses involving sustained reasoning.
- Individual questions may develop a single theme or be divided into unconnected parts.
- Questions may be presented in the form of words, symbols, diagrams or tables, or combinations of these.
- Normally, each question reflects an incline of difficulty, from relatively easy tasks at the start of a question to relatively difficult tasks at the end of a question. The emphasis is on problem solving.

# Paper 3 1 hr 20%

This paper consists of four sections, one on each of the options in the syllabus. Each section has a small number of extended-response questions based mainly on the option topic. Where possible, the first part of each question will be on core material leading to the option topic. When this is not readily achievable, as for example with the discrete mathematics option, the level of difficulty of the earlier part of a question will be comparable to that of the core questions.

Students must answer questions on one option topic only. Students must answer all the questions in the section chosen.

 

## Syllabus coverage

- Students must answer all the questions based on the option they have studied.
- Knowledge of the entire content of the option studied is required for this paper, as well as the core material.

## Question type

- Questions require extended responses involving sustained reasoning.
- Individual questions may develop a single theme or be divided into unconnected parts. Where this occurs, the unconnected parts will be clearly labelled as such.
- Questions may be presented in the form of words, symbols, diagrams or tables, or combinations of these.
- Normally, each question reflects an incline of difficulty, from relatively easy tasks at the start of a question to relatively difficult tasks at the end of a question. The emphasis is on problem solving.

## Mark allocation

- This paper is worth **60** marks, representing **20%** of the final mark. Approximately **15** marks are allocated to core material (or work of a similar level).
- Questions in this section may be unequal in terms of length and level of difficulty. Therefore, individual questions may not necessarily be worth the same number of marks. The exact number of marks allocated to each question is indicated at the start of each question. Each section is worth **60** marks, and the overall level of difficulty of each section should be the same.

# Guidelines

## Notation

Of the various notations in use, the IBO has chosen to adopt a system of notation based on the recommendations of the International Organization for Standardization (ISO). This notation is used in the examination papers for this course without explanation. If forms of notation other than those listed in this guide are used on a particular examination paper, they are defined within the question in which they appear.

Because students are required to recognize, though not necessarily use, IBO notation in examinations, it is recommended that teachers introduce students to this notation at the earliest opportunity. Students are **not** allowed access to information about this notation in the examinations.

In a small number of cases, students may need to use alternative forms of notation in their written answers. This is because not all forms of IBO notation can be directly transferred into handwritten form. For vectors in particular the IBO notation uses a bold, italic typeface that cannot adequately be transferred into handwritten form. In this case, teachers should advise candidates to use alternative forms of notation in their written work (for example, $\vec{x}$ , $\bar{x}$ or $\underset{\sim}{x}$ ).

Students must always use correct mathematical notation, not calculator notation.

 

| | |
|---|---|
| $\mathbb{N}$ | the set of positive integers and zero, $\{0, 1, 2, 3, ...\}$ |
| $\mathbb{Z}$ | the set of integers, $\{0, \pm 1, \pm 2, \pm 3, ...\}$ |
| $\mathbb{Z}^+$ | the set of positive integers, $\{1, 2, 3, ...\}$ |
| $\mathbb{Q}$ | the set of rational numbers |
| $\mathbb{Q}^+$ | the set of positive rational numbers, $\{x \mid x \in \mathbb{Q}, x > 0\}$ |
| $\mathbb{R}$ | the set of real numbers |
| $\mathbb{R}^+$ | the set of positive real numbers, $\{x \mid x \in \mathbb{R}, x > 0\}$ |
| $\mathbb{C}$ | the set of complex numbers, $\{a + \mathrm{i}b \mid a, b \in \mathbb{R}\}$ |
| $\mathrm{i}$ | $\sqrt{-1}$ |
| $z$ | a complex number |
| $z^*$ | the complex conjugate of $z$ |
| $\lvert z \rvert$ | the modulus of $z$ |
| $\arg z$ | the argument of $z$ |
| $\mathrm{Re}\, z$ | the real part of $z$ |
| $\mathrm{Im}\, z$ | the imaginary part of $z$ |
| $\{x_1, x_2, ...\}$ | the set with elements $x_1, x_2, ...$ |
| $n(A)$ | the number of elements in the finite set $A$ |
| $\{x \mid \quad\}$ | the set of all $x$ such that |
| $\in$ | is an element of |
| $\notin$ | is not an element of |
| $\varnothing$ | the empty (null) set |
| $U$ | the universal set |
| $\cup$ | union |
| $\cap$ | intersection |
| $\subset$ | is a proper subset of |

| | |
|---|---|
| $\subseteq$ | is a subset of |
| $A'$ | the complement of the set $A$ |
| $A\times B$ | the Cartesian product of sets $A$ and $B$ (that is, $A\times B=\{(a,b)\mid a\in A,\ b\in B\}$) |
| $a\mid b$ | $a$ divides $b$ |
| $a^{1/n}$, $\sqrt[n]{a}$ | $a$ to the power of $\frac{1}{n}$, $n^{\text{th}}$ root of $a$ (if $a\geq 0$ then $\sqrt[n]{a}\geq 0$) |
| $a^{1/2}$, $\sqrt{a}$ | $a$ to the power $\frac{1}{2}$, square root of $a$ (if $a\geq 0$ then $\sqrt{a}\geq 0$) |
| $\lvert x\rvert$ | the modulus or absolute value of $x$, that is $\begin{cases} x \text{ for } x\geq 0,\ x\in\mathbb{R} \\ -x \text{ for } x<0,\ x\in\mathbb{R}\end{cases}$ |
| $\equiv$ | identity |
| $\approx$ | is approximately equal to |
| $>$ | is greater than |
| $\geq$ | is greater than or equal to |
| $<$ | is less than |
| $\leq$ | is less than or equal to |
| $\ngtr$ | is not greater than |
| $\nless$ | is not less than |
| $[a,b]$ | the closed interval $a\leq x\leq b$ |
| $]a,b[$ | the open interval $a<x<b$ |
| $u_n$ | the $n^{\text{th}}$ term of a sequence or series |
| $d$ | the common difference of an arithmetic sequence |
| $r$ | the common ratio of a geometric sequence |
| $S_n$ | the sum of the first $n$ terms of a sequence, $u_1+u_2+\ldots+u_n$ |
| $S_\infty$ | the sum to infinity of a sequence, $u_1+u_2+\ldots$ |

| | |
|---|---|
| $\sum_{i=1}^{n} u_i$ | $u_1 + u_2 + ... + u_n$ |
| $\prod_{i=1}^{n} u_i$ | $u_1 \times u_2 \times ... \times u_n$ |
| $\binom{n}{r}$ | $\frac{n!}{r!(n-r)!}$ |
| $f : A \to B$ | $f$ is a function under which each element of set $A$ has an image in set $B$ |
| $f : x \mapsto y$ | $f$ is a function under which $x$ is mapped to $y$ |
| $f(x)$ | the image of $x$ under the function $f$ |
| $f^{-1}$ | the inverse function of the function $f$ |
| $f \circ g$ | the composite function of $f$ and $g$ |
| $\lim_{x \to a} f(x)$ | the limit of $f(x)$ as $x$ tends to $a$ |
| $\frac{\mathrm{d}y}{\mathrm{d}x}$ | the derivative of $y$ with respect to $x$ |
| $f'(x)$ | the derivative of $f(x)$ with respect to $x$ |
| $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$ | the second derivative of $y$ with respect to $x$ |
| $f''(x)$ | the second derivative of $f(x)$ with respect to $x$ |
| $\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$ | the $n^{\text{th}}$ derivative of $y$ with respect to $x$ |
| $f^{(n)}(x)$ | the $n^{\text{th}}$ derivative of $f(x)$ with respect to $x$ |
| $\int y \, \mathrm{d}x$ | the indefinite integral of $y$ with respect to $x$ |
| $\int_a^b y \, \mathrm{d}x$ | the definite integral of $y$ with respect to $x$ between the limits $x = a$ and $x = b$ |
| $\mathrm{e}^x$ | exponential function of $x$ |
| $\log_a x$ | logarithm to the base $a$ of $x$ |
| $\ln x$ | the natural logarithm of $x$, $\log_{\mathrm{e}} x$ |

| | |
|---|---|
| sin, cos, tan | the circular functions |
| arcsin, arccos, arctan | the inverse circular functions |
| csc, sec, cot | the reciprocal circular functions |
| $\mathrm{A}(x, y)$ | the point A in the plane with Cartesian coordinates $x$ and $y$ |
| $[\mathrm{AB}]$ | the line segment with end points A and B |
| AB | the length of $[\mathrm{AB}]$ |
| $(\mathrm{AB})$ | the line containing points A and B |
| $\hat{A}$ | the angle at A |
| $\mathrm{C\hat{A}B}$ | the angle between $[\mathrm{CA}]$ and $[\mathrm{AB}]$ |
| $\Delta\mathrm{ABC}$ | the triangle whose vertices are A, B and C |
| $\boldsymbol{v}$ | the vector $\boldsymbol{v}$ |
| $\overrightarrow{\mathrm{AB}}$ | the vector represented in magnitude and direction by the directed line segment from A to B |
| $\boldsymbol{a}$ | the position vector $\overrightarrow{\mathrm{OA}}$ |
| $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ | unit vectors in the directions of the Cartesian coordinate axes |
| $\lvert\boldsymbol{a}\rvert$ | the magnitude of $\boldsymbol{a}$ |
| $\lvert\overrightarrow{\mathrm{AB}}\rvert$ | the magnitude of $\overrightarrow{\mathrm{AB}}$ |
| $\boldsymbol{v} \cdot \boldsymbol{w}$ | the scalar product of $\boldsymbol{v}$ and $\boldsymbol{w}$ |
| $\boldsymbol{v} \times \boldsymbol{w}$ | the vector product of $\boldsymbol{v}$ and $\boldsymbol{w}$ |
| $\boldsymbol{A}^{-1}$ | the inverse of the non-singular matrix $\boldsymbol{A}$ |
| $\boldsymbol{A}^{\mathrm{T}}$ | the transpose of the matrix $\boldsymbol{A}$ |
| $\det \boldsymbol{A}$ | the determinant of the square matrix $\boldsymbol{A}$ |
| $\boldsymbol{I}$ | the identity matrix |
| $\mathrm{P}(A)$ | probability of event $A$ |

| | |
|---|---|
| $\mathrm{P}(A')$ | probability of the event "not $A$" |
| $\mathrm{P}(A \mid B)$ | probability of the event $A$ given $B$ |
| $x_1, x_2, \ldots$ | observations |
| $f_1, f_2, \ldots$ | frequencies with which the observations $x_1, x_2, \ldots$ occur |
| $\mathrm{P}_x$ | probability distribution function $\mathrm{P}(X = x)$ of the discrete random variable $X$ |
| $f(x)$ | probability density function of the continuous random variable $X$ |
| $F(x)$ | cumulative distribution function of the continuous random variable $X$ |
| $\mathrm{E}(X)$ | the expected value of the random variable $X$ |
| $\mathrm{Var}(X)$ | the variance of the random variable $X$ |
| $\mu$ | population mean |
| $\sigma^2$ | population variance, $\sigma^2 = \dfrac{\sum_{i=1}^{k} f_i (x_i - \mu)^2}{n}$, where $n = \sum_{i=1}^{k} f_i$ |
| $\sigma$ | population standard deviation |
| $\overline{x}$ | sample mean |
| $s_n^2$ | sample variance, $s_n^2 = \dfrac{\sum_{i=1}^{k} f_i (x_i - \overline{x})^2}{n}$, where $n = \sum_{i=1}^{k} f_i$ |
| $s_n$ | standard deviation of the sample |
| $s_{n-1}^2$ | unbiased estimate of the population variance, $s_{n-1}^2 = \dfrac{n}{n-1} s_n^2 = \dfrac{\sum_{i=1}^{k} f_i (x_i - \overline{x})^2}{n-1}$, where $n = \sum_{i=1}^{k} f_i$ |
| $\mathrm{B}(n,\ p)$ | binomial distribution with parameters $n$ and $p$ |
| $\mathrm{Po}(m)$ | Poisson distribution with mean $m$ |
| $\mathrm{N}(\mu, \sigma^2)$ | normal distribution with mean $\mu$ and variance $\sigma^2$ |

| | |
|---|---|
| $X \sim \mathrm{B}(n, p)$ | the random variable *X* has a binomial distribution with parameters *n* and *p* |
| $X \sim \mathrm{Po}(m)$ | the random variable *X* has a Poisson distribution with mean *m* |
| $X \sim \mathrm{N}(\mu, \sigma^2)$ | the random variable *X* has a normal distribution with mean $\mu$ and variance $\sigma^2$ |
| $\Phi$ | cumulative distribution function of the standardized normal variable with distribution $\mathrm{N}(0,1)$ |
| $\nu$ | number of degrees of freedom |
| $\chi^2$ | chi-squared distribution |
| $\chi^2_{calc}$ | the chi-squared test statistic, where $\chi^2_{calc} = \sum \frac{(f_o - f_e)^2}{f_e}$ |
| $A \setminus B$ | the difference of the sets *A* and *B* (that is, $A \setminus B = A \cap B' = \{x \mid x \in A \text{ and } x \notin B\}$) |
| $A \Delta B$ | the symmetric difference of the sets *A* and *B* (that is, $A \Delta B = (A \setminus B) \cup (B \setminus A)$) |
| $\kappa_n$ | a complete graph with *n* vertices |
| $\kappa_{n,m}$ | a complete bipartite graph with one set of *n* vertices and another set of *m* vertices |
| $\mathbb{Z}_p$ | the set of equivalence classes $\{0, 1, 2, \ldots, p-1\}$ of integers modulo *p* |
| $\gcd(a,b)$ | the greatest common divisor of integers *a* and *b* |
| $\mathrm{lcm}(a,b)$ | the least common multiple of integers *a* and *b* |
| $\boldsymbol{A}_G$ | the adjacency matrix of graph *G* |
| $\boldsymbol{C}_G$ | the cost adjacency matrix of graph *G* |

## Glossary of command terms

The following command terms are used without explanation on examination papers. Teachers should familiarize themselves and their students with the terms and their meanings. This list is not exhaustive. Other command terms may be used, but it should be assumed that they have their usual meaning (for example, "explain" and "estimate"). The terms included here are those that sometimes have a meaning in mathematics that is different from the usual meaning.

Further clarification and examples can be found in the teacher support material.

| | |
|---|---|
| *Write down* | Obtain the answer(s), usually by extracting information. Little or no calculation is required. Working does not need to be shown. |
| *Calculate* | Obtain the answer(s) showing all relevant working. "Find" and "determine" can also be used. |
| *Find* | Obtain the answer(s) showing all relevant working. "Calculate" and "determine" can also be used. |
| *Determine* | Obtain the answer(s) showing all relevant working. "Find" and "calculate" can also be used. |
| *Differentiate* | Obtain the derivative of a function. |
| *Integrate* | Obtain the integral of a function. |
| *Solve* | Obtain the solution(s) or root(s) of an equation. |
| *Draw* | Represent by means of a labelled, **accurate** diagram or graph, using a pencil. A ruler (straight edge) should be used for straight lines. Diagrams should be drawn to scale. Graphs should have points correctly plotted (if appropriate) and joined in a straight line or smooth curve. |
| *Sketch* | Represent by means of a diagram or graph, labelled if required. A sketch should give a general idea of the required shape of the diagram or graph. A sketch of a graph should include relevant features such as intercepts, maxima, minima, points of inflexion and asymptotes. |
| *Plot* | Mark the position of points on a diagram. |
| *Compare* | Describe the similarities and differences between two or more items. |
| *Deduce* | Show a result using known information. |
| *Justify* | Give a valid reason for an answer or conclusion. |
| *Prove* | Use a sequence of logical steps to obtain the required result in a formal way. |
| *Show that* | Obtain the required result (possibly using information given) without the formality of proof. "Show that" questions should not generally be "analysed" using a calculator. |
| *Hence* | Use the preceding work to obtain the required result. |
| *Hence or otherwise* | It is suggested that the preceding work is used, but other methods could also receive credit. |

## Weighting of objectives

Some objectives can be linked more easily to the different types of assessment. In particular, some will be assessed more appropriately in the internal assessment (as indicated in the following section) and only minimally in the examination papers.

| **Objective** | **Percentage weighting** |
|---|---|
| Know and use mathematical concepts and principles. | 15% |
| Read, interpret and solve a given problem using appropriate mathematical terms. | 15% |
| Organize and present information and data in tabular, graphical and/or diagrammatic forms. | 12% |
| Know and use appropriate notation and terminology (internal assessment). | 5% |
| Formulate a mathematical argument and communicate it clearly. | 10% |
| Select and use appropriate mathematical strategies and techniques. | 15% |
| Demonstrate an understanding of both the significance and the reasonableness of results (internal assessment). | 5% |
| Recognize patterns and structures in a variety of situations, and make generalizations (internal assessment). | 3% |
| Recognize and demonstrate an understanding of the practical applications of mathematics (internal assessment). | 3% |
| Use appropriate technological devices as mathematical tools (internal assessment). | 15% |
| Demonstrate an understanding of and the appropriate use of mathematical modelling (internal assessment). | 2% |

# Internal assessment details 20%

## The purpose of the portfolio

The purpose of the portfolio is to provide students with opportunities to be rewarded for mathematics carried out under ordinary conditions, that is, without the time limitations and pressure associated with written examinations. Consequently, the emphasis should be on good mathematical writing and thoughtful reflection.

The portfolio is also intended to provide students with opportunities to increase their understanding of mathematical concepts and processes. It is hoped that, by doing portfolio work, students benefit from these mathematical activities and find them both stimulating and rewarding.

The specific purposes of portfolio work are to:

- develop students' personal insight into the nature of mathematics and to develop their ability to ask their own questions about mathematics
- provide opportunities for students to complete extended pieces of mathematical work without the time constraints of an examination
- enable students to develop individual skills and techniques, and to allow them to experience the satisfaction of applying mathematical processes on their own
- provide students with the opportunity to experience for themselves the beauty, power and usefulness of mathematics
- provide students with the opportunity to discover, use and appreciate the power of a calculator or computer as a tool for doing mathematics
- enable students to develop the qualities of patience and persistence, and to reflect on the significance of the results they obtain
- provide opportunities for students to show, with confidence, what they know and what they can do.

## Objectives

The portfolio is internally assessed by the teacher and externally moderated by the IBO. Assessment criteria have been developed to relate to the mathematics objectives. In developing these criteria, particular attention has been given to the objectives listed here, since these cannot be easily addressed by means of timed, written examinations.

Where appropriate in the portfolio, students are expected to:

- know and use appropriate notation and terminology
- organize and present information and data in tabular, graphical and/or diagrammatic forms
- recognize patterns and structures in a variety of situations, and make generalizations
- demonstrate an understanding of and the appropriate use of mathematical modelling
- recognize and demonstrate an understanding of the practical applications of mathematics
- use appropriate technological devices as mathematical tools.

## Requirements

The portfolio must consist of two pieces of work assigned by the teacher and completed by the student during the course.

Each piece of student work contained in the portfolio must be based on:

- an area of the syllabus
- one of the two types of tasks
  - type I—mathematical investigation
  - type II—mathematical modelling.

The level of sophistication of the students' mathematical work should be similar to that contained in the syllabus. It is not intended that additional topics are taught to students to enable them to complete a particular task.

Each portfolio must contain two pieces of student work, each of the two types of task: the portfolio must contain one type I and one type II piece of work.

# Teaching considerations

These tasks should be completed at intervals throughout the course and should not be left until towards the end. Teachers are encouraged to allow students the opportunity to explore various aspects of as many different topics as possible.

Portfolio work should be integrated into the course of study so that it enhances student learning by introducing a topic, reinforcing mathematical meaning or taking the place of a revision exercise. Therefore, each task needs to correspond to the course of study devised by the individual teacher in terms of the knowledge and skills that the students have been taught.

## Use of technology

The need for proper **mathematical** notation and terminology, as opposed to **calculator** or **computer** notation must be stressed and reinforced, as well as adequate documentation of technology usage. Students will therefore be required to reflect on the mathematical processes and algorithms the technology is performing, and communicate them clearly and succinctly.

## Type I—mathematical investigation

While many teachers incorporate a problem-solving approach into their classroom practice, students also should be given the opportunity formally to carry out investigative work. The mathematical investigation is intended to highlight that:

- the idea of investigation is fundamental to the study of mathematics
- investigation work often leads to an appreciation of how mathematics can be applied to solve problems in a broad range of fields
- the discovery aspect of investigation work deepens understanding and provides intrinsic motivation
- during the process of investigation, students acquire mathematical knowledge, problem-solving techniques, a knowledge of fundamental concepts and an increase in self-confidence.

All investigations develop from an initial problem, the starting point. The problem must be clearly stated and contain no ambiguity. In addition, the problem should:

- provide a challenge and the opportunity for creativity
- contain multi-solution paths, that is, contain the potential for students to choose different courses of action from a range of options.

### Essential skills to be assessed

- Producing a strategy
- Generating data
- Recognizing patterns or structures
- Searching for further cases
- Forming a general statement
- Testing a general statement
- Justifying a general statement
- Appropriate use of technology

## Type II—Mathematical modelling

Problem solving usually elicits a process-oriented approach, whereas mathematical modelling requires an experimental approach. By considering different alternatives, students can use modelling to arrive at a specific conclusion, from which the problem can be solved. To focus on the actual process of modelling, the assessment should concentrate on the appropriateness of the model selected in relation to the given situation, and on a critical interpretation of the results of the model in the real-world situation chosen.

Mathematical modelling involves the following skills.

- Translating the real-world problem into mathematics
- Constructing a model
- Solving the problem
- Interpreting the solution in the real-world situation (that is, by the modification or amplification of the problem)
- Recognizing that different models may be used to solve the same problem
- Comparing different models
- Identifying ranges of validity of the models
- Identifying the possible limits of technology
- Manipulating data

### Essential skills to be assessed

- Identifying the problem variables
- Constructing relationships between these variables
- Manipulating data relevant to the problem
- Estimating the values of parameters within the model that cannot be measured or calculated from the data
- Evaluating the usefulness of the model
- Communicating the entire process
- Appropriate use of technology

# Follow-up and feedback

Teachers should ensure that students are aware of the significance of the results/conclusions that may be the outcome of a particular task. This is particularly important in the case when investigative work is used to introduce a topic on the syllabus. Teachers should allow class time for follow-up work when developing the course of study.

Students should also receive feedback on their own work so that they are aware of alternative strategies for developing their mathematical thinking and are provided with guidance for improving their skills in writing mathematics.