Assessment and the GDC

What students should write down in examinations

The current group 5 mathematics objectives state that students should "organize and present information and data in tabular, graphical and/or diagrammatic forms", and "formulate a mathematical argument and communicate it clearly". This means that it is important for students to learn to communicate effectively in examinations.

The assessment model has changed from previous mathematics courses, and students are now expected to show their working on all papers to achieve full marks. For mathematics HL and SL paper 1, it is no longer the case that full marks will be awarded for providing the correct answer only. To receive full marks on any question, the correct answer will generally need to be supported by suitable working.

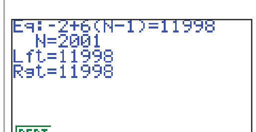
The discussions about what students should write down in examinations have been going on for a long time, even before the advent of the GDC. The answers to the questions "What should be written down in an examination when I have used a calculator?" and "How do I show my working?" also apply to situations when a calculator is not used. The important factor is good communication.

Example question 1

In an arithmetic sequence, the first term is –2, the fourth term is 16, and the nth term is 11,998.

(a) Find the common difference d.

(b) Find the value of n. **[6 marks]**

Write down	Rationale
$u_1 = -2, u_4 = 16, u_n = 11998$ $u_n = u_1 + (n-1)d$	Write down the given information in mathematical language and write down any relevant formula.
(a) $16 = -2 + 3d$ (This gives $d = 6$) (b) $11998 = -2 + (n-1)6$	Set up the equations.
	Use an equation solver (this example uses the Casio 9850+) to compute. 
(a) $d = 6$ (b) $n = 2001$	Write down the answer. Check that the answer matches the question asked and contains no errors.

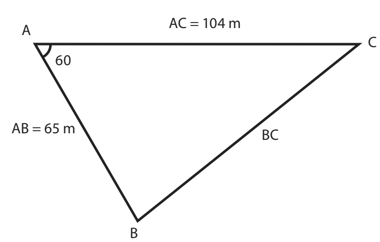
The first step in answering most questions is to extract the information, select an appropriate strategy, and then use the information. Quite often, this involves rearranging the information into a form that is suitable for use. When using a GDC, the information may need to be written in a form suitable for input into the GDC—what some teachers call "getting the question calculator ready". For example, if asked to find the area between curves, students may need to identify the points of intersection and make a note of these. They should also communicate that they are using a definite integral between appropriate limits to find the area. The first part of a question may ask them to write down an integral representing the area but, even if it does not, this should be one of the first steps. Therefore, in the first part of the answer students should explain in mathematical language (not calculator notation) what they are doing.

Students need to give enough information so that the important steps in the solution are apparent. However, it is not necessary for them to write down every single algebraic or arithmetic step. Copying all results from the calculator onto paper would interrupt a chain of thought, be time-consuming and probably increase the likelihood of errors occurring. The challenge is to determine what constitutes an appropriate solution. Teachers should encourage students to identify "key features" of solutions, and make sure they write these down.

Over the last few years, discussions between examiners and teachers attending meetings at the International Baccalaureate Curriculum and Assessment Centre (IBCA) have highlighted that there are many different approaches possible. There are innovative and interesting ways of using a GDC to answer questions that most people would have thought could not be done on a GDC. Various people have been asked to share their thoughts, and some of these appear in this document.

Example question 2

A farmer owns a triangular field ABC. The side AC is 104m, the side AB is 65m and the angle between these two sides is 60°. Calculate the length of the third side of the field.

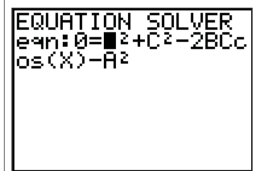
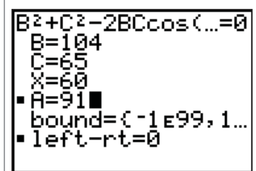
Write down	Rationale
	Draw an appropriate diagram.
$a^2 = b^2 + c^2 - 2bc\cos A$	Identify and write down the appropriate rule to be used (cosine rule).

Two likely methods are possible.

Method 1

Write down	Rationale
$\text{BC}^2 = 65^2 + 104^2 - 2(65)(104)\cos 60^\circ$	Substitute in rule.
$\text{BC}^2 = 8281$	Compute.
$\text{BC} = 91\text{m}$	Write down the answer.

Method 2

Write down	Rationale
$0 = b^2 + c^2 - 2bc\cos A - a^2$	Rewrite the rule equal to zero and enter into the equation solver.  (TI-84+SE)
BC = 91m	Highlight A and solve.  Write down the answer.

Note that there is generally no "right" or "wrong" use of the GDC. Some questions are written so that they can only be answered using a GDC; some can be answered with or without a GDC, and others are meant to be done analytically.

In particular, it is not appropriate to use a GDC for a question that asks for an exact answer or uses the command term "show that".

"Show that" and answer "carried forward" = find questions

The style of setting several parts to a question often requires students to "carry forward" an answer from one part of the question to another. This "carry forward" plays an important role in the remainder of a question; the answer is often provided and the command term used is "show that".

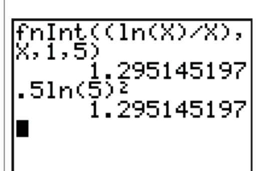
In this situation, teachers should advise students to treat it as a "find" question even though the answer is given. It is a good idea for students to use the given answer as a check that they have written down the question correctly. Even if they fail to do the "show that" part, they should still use the given answer in subsequent parts of a question. The examples below demonstrate what students should write down when answering this type of question and when it may be appropriate for them to use a GDC.

Example question 1

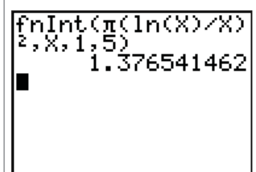
The function f is defined on the domain $x \geq 1$ by $f(x) = \dfrac{\ln x}{x}$. Let R be the region enclosed by the graph of f, the x-axis and the line $x = 5$.

(a) Find the **exact** value of the area of R.

(b) The region R is rotated through an angle of 2π about the x-axis. Find the volume of the solid of revolution generated.

Write down	**Rationale**
(a) Area = $\int_1^5 \frac{\ln x}{x}\,\mathrm{d}x$	Write down an appropriate mathematical formula representing the area.
Using the GDC to compute the integral would be inappropriate in this part since the question demands the exact value. An appropriate use of the GDC might be to check the answer.	
$u = \ln x, \mathrm{d}u = \frac{1}{x}\mathrm{d}x$ $\int u\mathrm{d}u = \frac{u^2}{2}\left(= \frac{(\ln x)^2}{2}\right)$ Area = $\left[\frac{(\ln x)^2}{2}\right]_1^5 = \frac{1}{2}\left((\ln 5)^2 - (\ln 1)^2\right)$ Area = $\frac{1}{2}(\ln 5)^2$	Either Find the integral by substitution/ inspection.
$u = \ln x, \mathrm{d}v = \frac{1}{x} \Rightarrow \mathrm{d}u = \frac{1}{x}, v = \ln x$ $I = uv - \int u\mathrm{d}v = (\ln x)^2 - \int \ln x \frac{1}{x}\mathrm{d}x = (\ln x)^2 - I$ $\Rightarrow 2I = (\ln x)^2 \Rightarrow I = \frac{(\ln x)^2}{2}$ $\Rightarrow \text{area} = \left[\frac{(\ln x)^2}{2}\right]_1^5 = \frac{1}{2}\left((\ln 5)^2 - (\ln 1)^2\right)$ Area = $\frac{1}{2}(\ln 5)^2$	Or Find the integral I by parts.
	The appropriate use of the GDC in this example might be to check the answer. 

In part (b), the GDC is appropriate as a computational tool for the definite integration.

Write down	Rationale
(b) $V=\int_a^b \pi y^2 \, \mathrm{d}x$	Write down an appropriate mathematical formula representing the volume.
$=\int_1^5 \pi\left(\frac{\ln x}{x}\right)^2 \mathrm{d}x$	Write the integration with the values given within the problem.
	Use the GDC to calculate the definite integral. 
$= 1.38$	Write down the answer.

The GDC also provides the opportunity for highly original and sometimes very unexpected solutions.

Example question 2

The continuous random variable X has probability density function:

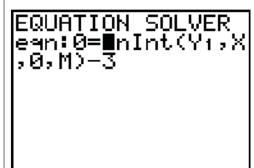
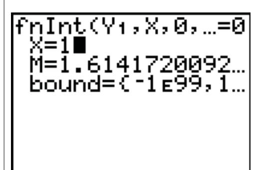
$f(x)=\frac{1}{6}x(1+x^2)$ for $0 \le x \le 2$,

$f(x)=0$ otherwise.

Find the median of X.

Write down	Rationale
The median m satisfies $\frac{1}{6}\int_0^m (x+x^3)\mathrm{d}x=\frac{1}{2}$	Write down an appropriate mathematical formula representing the median.
$\frac{m^2}{2}+\frac{m^4}{4}=3$ $\Rightarrow m^4+2m^2-12=0$ $m^2=\frac{-2\pm\sqrt{4+48}}{2}=2.60555...$	One method would be to evaluate the integral and solve it algebraically.
$m = 1.61$	Write down the answer.

An alternative is to use the equation solver. Note that the first step of writing down a mathematical formula does not change, even when using the GDC for the majority of working.

Write down	**Rationale**
The median m satisfies $\frac{1}{6}\int_0^m (x+x^3)\mathrm{d}x = \frac{1}{2}$	Write down an appropriate mathematical formula representing the median.
$\int_0^m (x+x^3)\mathrm{d}x - 3 = 0$	Simplify and rewrite equal to zero.
	Enter $x + x^3$ as Y1. Then enter the expression into the equation solver (shown below on the TI-84+SE). 
	Highlight and solve for M. 
$m = 1.61$	Write down the answer.

