SYLLABUS DETAILS

Format of the syllabus

The syllabus to be taught is presented as three columns.

- **Content**: the first column lists, under each topic, the sub-topics to be covered.
- **Amplifications/inclusions**: the second column contains more explicit information on specific sub-topics listed in the first column. This helps to define what is required in terms of preparing for the examination.
- **Exclusions**: the third column contains information about what is not required in terms of preparing for the examination.

Although the mathematics HL course is similar in content to parts of the mathematics SL course, there are differences. In particular, students and teachers are expected to take a more sophisticated approach for mathematics HL, during the course and in the examinations. Where appropriate, guidelines are provided in the second and third columns of the syllabus details (as indicated by the phrase "See SL guide").

Teaching notes and calculator suggestions linked to the syllabus content are contained in a separate publication.

Course of study

Teachers are required to teach all the sub-topics listed for the seven topics in the core, together with all the sub-topics in the chosen option.

The topics in the syllabus do not need to be taught in the order in which they appear in this guide. Similarly, it is not necessary to teach all the topics in the core before starting to teach an option. Teachers should therefore construct a course of study that is tailored to the needs of their students and that integrates the areas covered by the syllabus, and, where necessary, the presumed knowledge (PK).

Integration of portfolio assignments

The two pieces of work for the portfolio, based on the two types of tasks (mathematical investigation and mathematical modelling), should be incorporated into the course of study, and should relate directly to topics in the syllabus. Full details of how to do this are given in the section on internal assessment.

Time allocation

The recommended teaching time for higher level courses is 240 hours. For mathematics HL, it is expected that 10 hours will be spent on work for the portfolio. The time allocations given in this guide are approximate, and are intended to suggest how the remaining 230 hours allowed for the teaching of the syllabus might be allocated. However, the exact time spent on each topic depends on a number of factors, including the background knowledge and level of preparedness of each student. Teachers should therefore adjust these timings to correspond to the needs of their students.

Use of calculators

Students are expected to have access to a graphic display calculator (GDC) at all times during the course. The minimum requirements are reviewed as technology advances, and updated information will be provided to schools. It is expected that teachers and schools monitor calculator use with reference to the calculator policy. Regulations covering the types of calculator allowed are provided in the *Vade Mecum*. Further information and advice is provided in the teacher support material.

There are specific requirements for calculators used by students studying the statistics and probability option.

Mathematics HL information booklet

Because each student is required to have access to a clean copy of this booklet during the examination, it is recommended that teachers ensure students are familiar with the contents of this document from the beginning of the course. The booklet is provided by the IBO and is published separately.

Teacher support materials

A variety of teacher support materials will accompany this guide. These materials will include suggestions to help teachers integrate the use of GDCs into their teaching, guidance for teachers on the marking of portfolios, and specimen examination papers and markschemes. These will be distributed to all schools.

External assessment guidelines

It is recommended that teachers familiarize themselves with the section on external assessment guidelines, as this contains important information about the examination papers. In particular, students need to be familiar with notation the IBO uses and the command terms, as these will be used without explanation in the examination papers.

Presumed knowledge

General

Students are not required to be familiar with all the topics listed as PK **before** they start this course. However, they should be familiar with these topics before they take the **examinations**, because questions assume knowledge of them.

Teachers must therefore ensure that any topics designated as PK that are unknown to their students at the start of the course are included at an early stage. They should also take into account the existing mathematical knowledge of their students to design an appropriate course of study for mathematics HL.

This list of topics is not designed to represent the outline of a course that might lead to the mathematics HL course. Instead, it lists the knowledge, together with the syllabus content, that is essential to successful completion of the mathematics HL course

Students must be familiar with SI (*Système International*) units of length, mass and time, and their derived units.

Topics

Number and algebra

Routine use of addition, subtraction, multiplication and division using integers, decimals and fractions, including order of operations.

Example: $2(3+4\times7)=62$.

Simple positive exponents.

Examples: $2^3=8$; $(-3)^3=-27$; $(-2)^4=16$.

Simplification of expressions involving roots (surds or radicals).

Examples: $\sqrt{27}+\sqrt{75}=8\sqrt{3}$; $\sqrt{3}\times\sqrt{5}=\sqrt{15}$.

Prime numbers and factors, including greatest common factors and least common multiples.

Simple applications of ratio, percentage and proportion, linked to similarity.

Definition and elementary treatment of absolute value (modulus), $|a|$.

Rounding, decimal approximations and significant figures, including appreciation of errors.

Expression of numbers in standard form (scientific notation), that is, $a\times10^k$, $1\le a<10$, $k\in\mathbb{Z}$.

Concept and notation of sets, elements, universal (reference) set, empty (null) set, complement, subset, equality of sets, disjoint sets. Operations on sets: union and intersection. Commutative, associative and distributive properties. Venn diagrams.

Number systems: natural numbers; integers, $\mathbb{Z}$; rationals, $\mathbb{Q}$, and irrationals; real numbers, $\mathbb{R}$.

Intervals on the real number line using set notation and using inequalities. Expressing the solution set of a linear inequality on the number line and in set notation.

The concept of a relation between the elements of one set and between the elements of one set and those of another set. Mappings of the elements of one set onto or into another, or the same, set. Illustration by means of tables, diagrams and graphs.

Basic manipulation of simple algebraic expressions involving factorization and expansion.

Examples: $ab+ac=a(b+c)$; $(a\pm b)^2=a^2+b^2\pm2ab$; $a^2-b^2=(a-b)(a+b)$;
$3x^2+5x+2=(3x+2)(x+1)$; $xa-2a+xb-2b=(x-2)(a+b)$.

Rearrangement, evaluation and combination of simple formulae. Examples from other subject areas, particularly the sciences, should be included.

The linear function $x\mapsto ax+b$ and its graph, gradient and y-intercept.

Addition and subtraction of algebraic fractions with denominators of the form $ax+b$.

Example: $\frac{2x}{3x-1}+\frac{3x+1}{2x+4}$.

The properties of order relations: $<$, $\le$, $>$, $\ge$.

Examples: $a>b, c>0 \Rightarrow ac>bc$; $a>b, c<0 \Rightarrow ac<bc$.

Solution of equations and inequalities in one variable, including cases with rational coefficients.

Example: $\frac{3}{7}-\frac{2x}{5}=\frac{1}{2}(1-x)\Rightarrow x=\frac{5}{7}$.

Solution of simultaneous equations in two variables.

Geometry

Elementary geometry of the plane including the concepts of dimension for point, line, plane and space. Parallel and perpendicular lines, including $m_1 = m_2$, and $m_1 m_2 = -1$. Geometry of simple plane figures. The function $x \mapsto ax + b$: its graph, gradient and y-intercept.

Angle measurement in degrees. Compass directions and bearings. Right-angle trigonometry. Simple applications for solving triangles.

Pythagoras' theorem and its converse.

The Cartesian plane: ordered pairs (x, y), origin, axes. Mid-point of a line segment and distance between two points in the Cartesian plane.

Simple geometric transformations: translation, reflection, rotation, enlargement. Congruence and similarity, including the concept of scale factor of an enlargement.

The circle, its centre and radius, area and circumference. The terms "arc", "sector", "chord", "tangent" and "segment".

Perimeter and area of plane figures. Triangles and quadrilaterals, including parallelograms, rhombuses, rectangles, squares, kites and trapeziums (trapezoids); compound shapes.

Statistics

Descriptive statistics: collection of raw data, display of data in pictorial and diagrammatic forms (for example, pie charts, pictograms, stem and leaf diagrams, bar graphs and line graphs).

Calculation of simple statistics from discrete data, including mean, median and mode.

Core syllabus content

Topic 1—Core: Algebra — 20 hrs

Aims

The aim of this section is to introduce students to some basic algebraic concepts and applications.

Details

	Content	Amplifications/inclusions	Exclusions
1.1	Arithmetic sequences and series; sum of finite arithmetic series; geometric sequences and series; sum of finite and infinite geometric series. Sigma notation.	Examples of applications: compound interest and population growth.	
1.2	Exponents and logarithms. Laws of exponents; laws of logarithms. Change of base.	Elementary treatment only is required. $\log_b a = \frac{\log_c a}{\log_c b}$.	
1.3	Counting principles, including permutations and combinations. The binomial theorem: expansion of $(a+b)^n$, $n \in \mathbb{N}$.	Simple applications only. The formula for $\binom{n}{r}$ also denoted by ${}^n\mathrm{C}_r$.	Formula for ${}^n\mathrm{P}_r$. Permutations where some objects are identical. **See SL guide**

Topic 1—Core: Algebra (continued)

	Content	Amplifications/inclusions	Exclusions
1.4	Proof by mathematical induction. Forming conjectures to be proved by mathematical induction.		Proof of binomial theorem.
1.5	Complex numbers: the number $\mathrm{i} = \sqrt{-1}$; the terms real part, imaginary part, conjugate, modulus and argument. Cartesian form $z = a + \mathrm{i}b$. Modulus–argument form $z = r(\cos\theta + \mathrm{i}\sin\theta)$. The complex plane.	Awareness that $z = r(\cos\theta + \mathrm{i}\sin\theta)$ can be written as $z = r\mathrm{e}^{\mathrm{i}\theta}$ and $z = r\,\mathrm{cis}\,\theta$. The complex plane is also known as the Argand diagram.	Loci in the complex plane.
1.6	Sums, products and quotients of complex numbers.		
1.7	De Moivre's theorem. Powers and roots of a complex number.	Proof by mathematical induction for $n \in \mathbb{Z}^+$.	
1.8	Conjugate roots of polynomial equations with real coefficients.		Equations with complex coefficients.

Topic 2—Core: Functions and equations 26 hrs

Aims

The aims of this section are to explore the notion of function as a unifying theme in mathematics, and to apply functional methods to a variety of mathematical situations. It is expected that extensive use will be made of a GDC in both the development and the application of this topic.

Details

	Content	Amplifications/inclusions	Exclusions
2.1	Concept of function $f : x \mapsto f(x)$: domain, range; image (value).	On examination papers: if the domain is the set of real numbers then the statement "$x \in \mathbb{R}$" will be omitted.	The term "codomain".
	Composite functions $f \circ g$; identity function.	The composite function $(f \circ g)(x)$ is defined as $f(g(x))$.	
	Inverse function f^{-1}.	Distinction between one-to-one and many-to-one functions. Domain restriction.	**See SL guide**
2.2	The graph of a function; its equation $y = f(x)$.	On examination papers: questions may be set that require the graphing of functions that do not explicitly appear on the syllabus.	
	Function graphing skills:		
	use of a GDC to graph a variety of functions		
	investigation of key features of graphs	Identification of asymptotes.	
	solutions of equations graphically.	May be referred to as roots of equations, or zeros of functions.	

Topic 2—Core: Functions and equations (continued)

	Content	Amplifications/inclusions	Exclusions
2.3	Transformations of graphs: translations; stretches; reflections in the axes.	Translations: $y = f(x) + b$; $y = f(x - a)$. Stretches: $y = pf(x)$; $y = f(x/q)$. Reflections (in both axes): $y = -f(x)$; $y = f(-x)$. Examples: $y = x^2$ used to obtain $y = 3x^2 + 2$ by a stretch of scale factor 3 in the y-direction followed by a translation of $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$. $y = \sin x$ used to obtain $y = 3\sin 2x$ by a stretch of scale factor 3 in the y-direction and a stretch of scale factor $\frac{1}{2}$ in the x-direction.	
	The graph of $y = f^{-1}(x)$ as the reflection in the line $y = x$ of the graph of $y = f(x)$. The graph of $y = \frac{1}{f(x)}$ from $y = f(x)$. The graphs of the absolute value functions, $y = \lvert f(x) \rvert$ and $y = f(\lvert x \rvert)$.		
2.4	The reciprocal function $x \mapsto \frac{1}{x}$, $x \neq 0$: its graph; its self-inverse nature.		

Topic 2—Core: Functions and equations (continued)

	Content	Amplifications/inclusions	Exclusions
2.5	The quadratic function $x \mapsto ax^2 + bx + c$: its graph. Axis of symmetry $x = -\frac{b}{2a}$. The form $x \mapsto a(x-h)^2 + k$. The form $x \mapsto a(x-p)(x-q)$.	Real coefficients only.	
2.6	The solution of $ax^2 + bx + c = 0,\ a \neq 0$. The quadratic formula. Use of the discriminant $\Delta = b^2 - 4ac$.		On examination papers: questions requiring elaborate factorization techniques will not be set.
2.7	The function: $x \mapsto a^x$, $a > 0$. The inverse function $x \mapsto \log_a x$, $x > 0$. Graphs of $y = a^x$ and $y = \log_a x$. Solution of $a^x = b$ using logarithms.	$\log_a a^x = x$; $a^{\log_a x} = x$, $x > 0$.	

Topic 2—Core: Functions and equations (continued)

	Content	Amplifications/inclusions	Exclusions
2.8	The exponential function $x \mapsto e^x$. The logarithmic function $x \mapsto \ln x$, $x > 0$.	 $a^x = e^{x \ln a}$. Examples of applications: compound interest, growth and decay.	
2.9	Inequalities in one variable, using their graphical representation. Solution of $g(x) \geq f(x)$, where *f*, *g* are linear or quadratic.	Use of the absolute value sign in inequalities. Analytical solution for simple cases.	On examination papers: questions requiring elaborate manipulation will not be set.
2.10	Polynomial functions. The factor and remainder theorems, with application to the solution of polynomial equations and inequalities.	The graphical significance of repeated roots.	

Topic 3—Core: Circular functions and trigonometry 22 hrs

Aims

The aims of this section are to explore the circular functions, to introduce some important trigonometric identities and to solve triangles using trigonometry.

Details

	Content	Amplifications/inclusions	Exclusions
3.1	The circle: radian measure of angles; length of an arc; area of a sector.	Radian measure may be expressed as multiples of π, or decimals.	
3.2	Definition of $\cos\theta$ and $\sin\theta$ in terms of the unit circle. Definition of $\tan\theta$ as $\frac{\sin\theta}{\cos\theta}$. Definition of $\sec\theta$, $\csc\theta$ and $\cot\theta$. Pythagorean identities: $\cos^2\theta+\sin^2\theta=1$; $1+\tan^2\theta=\sec^2\theta$; $1+\cot^2\theta=\csc^2\theta$.		**See SL guide**
3.3	Compound angle identities. Double angle identities.	Proof of compound angle identities: $\sin(A\pm B)$, $\cos(A\pm B)$, $\tan(A\pm B)$. Proof of double angle identities. Given $\sin\theta$, finding possible values of other ratios (for example $\sin 2\theta$) without finding θ.	**See SL guide**

Topic 3—Core: Circular functions and trigonometry (continued)

	Content	Amplifications/inclusions	Exclusions
3.4	The circular functions $\sin x$, $\cos x$ and $\tan x$; their domains and ranges; their periodic nature; their graphs.	On examination papers: radian measure should be assumed unless otherwise indicated, for example, by $x \mapsto \sin x°$.	
	Composite functions of the form $f(x) = a\sin(b(x+c)) + d$.	Example: $f(x) = 3\tan(4(x-2)) + 1$.	
	The inverse functions $x \mapsto \arcsin x$, $x \mapsto \arccos x$, $x \mapsto \arctan x$; their domains and ranges; their graphs.		On examination papers: questions requiring elaborate analytical treatment of inverse trigonometric functions will not be set.
		Examples of applications: height of tide; Ferris wheel.	**See SL guide**
3.5	Solution of trigonometric equations in a finite interval.	Examples: $2\sin x = 3\cos x$, $0 \le x \le 2\pi$. $2\sin 2x = 3\cos x$, $0° \le x \le 180°$. $2\sin x = \cos 2x$, $-\pi \le x \le \pi$.	The general solution of trigonometric equations.
	Use of trigonometric identities and factorization to transform equations.	Both analytical and graphical methods required.	

Topic 3—Core: Circular functions and trigonometry (continued)

	Content	Amplifications/inclusions	Exclusions
3.6	Solution of triangles.		
	The cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$.		
	The sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.	The ambiguous case of the sine rule.	
	Area of a triangle as $\frac{1}{2}ab\sin C$.		
		Applications to real-life situations in two dimensions, and simple cases in three dimensions, for example, navigation.	

Topic 4—Core: Matrices 12 hrs

Aims

The aim of this section is to provide an elementary introduction to matrices, a fundamental concept of linear algebra.

Details

	Content	Amplifications/inclusions	Exclusions
4.1	Definition of a matrix: the terms element, row, column and order.	Use of matrices to store data.	Use of matrices to represent transformations.
4.2	Algebra of matrices: equality; addition; subtraction; multiplication by a scalar.	Matrix operations to handle or process information.	
	Multiplication of matrices.		
	Identity and zero matrices.		
4.3	Determinant of a square matrix.	The terms singular and non-singular matrices.	
	Calculation of 2×2 and 3×3 determinants.	The result $\det \boldsymbol{AB} = \det \boldsymbol{A} \det \boldsymbol{B}$.	Cofactors and minors.
	Inverse of a matrix: conditions for its existence.	Obtaining the inverse of a 3×3 matrix using a GDC.	Other methods for finding the inverse of a 3×3 matrix.
4.4	Solution of systems of linear equations (a maximum of three equations in three unknowns).		
	Conditions for the existence of a unique solution, no solution and an infinity of solutions.	These cases can be investigated using row reduction, including the use of augmented matrices. Unique solutions can also be found using inverse matrices.	

Topic 5—Core: Vectors 22 hrs

Aims

The aim of this section is to introduce the use of vectors in two and three dimensions, and to facilitate solving problems involving points, lines and planes.

Details

	Content	Amplifications/inclusions	Exclusions
5.1	Vectors as displacements in the plane and in three dimensions.	Distance between points in three dimensions.	
	Components of a vector; column representation $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = v_1\boldsymbol{i} + v_2\boldsymbol{j} + v_3\boldsymbol{k}$.	Components are with respect to the unit vectors $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ (standard basis).	
	Algebraic and geometric approaches to the following topics:		
	the sum and difference of two vectors; the zero vector, the vector $-\boldsymbol{v}$;	The difference of $\boldsymbol{v}$ and $\boldsymbol{w}$ is $\boldsymbol{v} - \boldsymbol{w} = \boldsymbol{v} + (-\boldsymbol{w})$.	
	multiplication by a scalar, $k\boldsymbol{v}$;		
	magnitude of a vector, $\lvert\boldsymbol{v}\rvert$;		
	unit vectors; base vectors $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$;		
	position vectors $\overrightarrow{\mathrm{OA}} = \boldsymbol{a}$.	$\overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{OB}} - \overrightarrow{\mathrm{OA}} = \boldsymbol{b} - \boldsymbol{a}$.	

Topic 5—Core: Vectors (continued)

	Content	Amplifications/inclusions	Exclusions
5.2	The scalar product of two vectors, $\boldsymbol{v} \cdot \boldsymbol{w} = \lvert\boldsymbol{v}\rvert\lvert\boldsymbol{w}\rvert\cos\theta$; $\boldsymbol{v} \cdot \boldsymbol{w} = v_1w_1 + v_2w_2 + v_3w_3$. Algebraic properties of the scalar product. Perpendicular vectors; parallel vectors. The angle between two vectors.	The scalar product is also known as the "dot product" or "inner product". For non-zero perpendicular vectors $\boldsymbol{v} \cdot \boldsymbol{w} = 0$; for non-zero parallel vectors $\boldsymbol{v} \cdot \boldsymbol{w} = \pm\lvert\boldsymbol{v}\rvert\lvert\boldsymbol{w}\rvert$.	Projections.
5.3	Vector equation of a line $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}$. The angle between two lines.	Lines in the plane and in three-dimensional space. Knowledge of the following forms for equations of lines. Parametric form: $x = x_0 + \lambda l$, $y = y_0 + \lambda m$, $z = z_0 + \lambda n$. Cartesian form: $\frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n}$.	**See SL guide**
5.4	Coincident, parallel, intersecting and skew lines, distinguishing between these cases. Points of intersection.		

Topic 5—Core: Vectors (continued)

	Content	Amplifications/inclusions	Exclusions
5.5	The vector product of two vectors, $\boldsymbol{v} \times \boldsymbol{w}$. The determinant representation. Geometric interpretation of $\lvert \boldsymbol{v} \times \boldsymbol{w} \rvert$.	The vector product is also known as the cross product. Areas of triangles and parallelograms.	
5.6	Vector equation of a plane $\boldsymbol{r} = \boldsymbol{a} + \lambda \boldsymbol{b} + \mu \boldsymbol{c}$. Use of normal vector to obtain the form $\boldsymbol{r} \cdot \boldsymbol{n} = \boldsymbol{a} \cdot \boldsymbol{n}$. Cartesian equation of a plane $ax + by + cz = d$.		
5.7	Intersections of: a line with a plane; two planes; three planes. Angle between: a line and a plane; two planes.	Inverse matrix method and row reduction for finding the intersection of three planes. Awareness that three planes may intersect in a point, or in a line, or not at all.	

Topic 6—Core: Statistics and probability 40 hrs

Aims

The aim of this section is to introduce basic concepts. It may be considered as three parts: manipulation and presentation of statistical data (6.1–6.4), the laws of probability (6.5–6.8), and random variables and their probability distributions (6.9–6.11). It is expected that most of the calculations required will be done on a GDC. The emphasis is on understanding and interpreting the results obtained.

Details

	Content	Amplifications/inclusions	Exclusions
6.1	Concepts of population, sample, random sample and frequency distribution of discrete and continuous data.	Elementary treatment only.	
6.2	Presentation of data: frequency tables and diagrams, box and whisker plots.	Treatment of both continuous and discrete data.	
	Grouped data: mid-interval values, interval width, upper and lower interval boundaries,		
	frequency histograms.	A frequency histogram uses equal class intervals.	Histograms based on unequal class intervals.

Topic 6—Core: Statistics and probability (continued)

	Content	Amplifications/inclusions	Exclusions
6.3	Mean, median, mode; quartiles, percentiles. Range; interquartile range; variance, standard deviation.	Awareness that the population mean, μ, is generally unknown, and that the sample mean, $\bar{x}$, serves as an unbiased estimate of this quantity. Awareness of the concept of dispersion and an understanding of the significance of the numerical value of the standard deviation. Obtain the standard deviation (and indirectly the variance) from a GDC and by other methods. Awareness that the population variance, σ^2, is generally unknown, and that $s_{n-1}^2 = \frac{n}{n-1} s_n^2$ serves as an unbiased estimate of σ^2.	Estimation of the mode from a histogram. Formal treatment of unbiased estimation. **See SL guide**
6.4	Cumulative frequency; cumulative frequency graphs; use to find median, quartiles, percentiles.		
6.5	Concepts of trial, outcome, equally likely outcomes, sample space (U) and event. The probability of an event A as $\mathrm{P}(A) = \frac{n(A)}{n(U)}$. The complementary events A and A' (not A); $\mathrm{P}(A) + \mathrm{P}(A') = 1$.	The calculation of $n(A)$ and $n(U)$ may involve counting principles.	

Topic 6—Core: Statistics and probability (continued)

	Content	Amplifications/inclusions	Exclusions
6.6	Combined events, the formula: $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$. $\mathrm{P}(A \cap B) = 0$ for mutually exclusive events.	Appreciation of the non-exclusivity of "or". Use of $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$ for mutually exclusive events.	
6.7	Conditional probability; the definition: $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$. Independent events; the definition: $\mathrm{P}(A \mid B) = \mathrm{P}(A) = \mathrm{P}(A \mid B')$. Use of Bayes' theorem for two events.	The term "independent" is equivalent to "statistically independent". Use of $\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B)$ for independent events. $\mathrm{P}(B \mid A) = \dfrac{\mathrm{P}(B)\mathrm{P}(A \mid B)}{\mathrm{P}(B)\mathrm{P}(A \mid B) + \mathrm{P}(B')\mathrm{P}(A \mid B')}$.	
6.8	Use of Venn diagrams, tree diagrams and tables of outcomes to solve problems.		

Topic 6—Core: Statistics and probability (continued)

	Content	Amplifications/inclusions	Exclusions
6.9	Concept of discrete and continuous random variables and their probability distributions. Definition and use of probability density functions. Expected value (mean), mode, median, variance and standard deviation.	Knowledge and use of the formulae for $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. Applications of expectations, for example, games of chance.	**See SL guide**
6.10	Binomial distribution, its mean and variance. Poisson distribution, its mean and variance.	Conditions under which random variables have these distributions.	Formal proof of means and variances. **See SL guide**
6.11	Normal distribution. Properties of the normal distribution. Standardization of normal variables.	Appreciation that the standardized value (z) gives the number of standard deviations from the mean. Use of calculator (or tables) to find normal probabilities; the reverse process.	Normal approximation to the binomial distribution.

Topic 7—Core: Calculus 48 hrs

Aims

The aim of this section is to introduce students to the basic concepts and techniques of differential and integral calculus and their application.

Details

	Content	Amplifications/inclusions	Exclusions
7.1	Informal ideas of limit and convergence.	Only an informal treatment of limit and convergence, including the result $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$.	
	Definition of derivative as $f'(x) = \lim_{h \to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$.	Use of this definition for differentiation of polynomials, and for justification of other derivatives. Familiarity with both forms of notation, $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $f'(x)$, for the first derivative.	
	Derivative of x^n $(n \in \mathbb{Q})$, $\sin x$, $\cos x$, $\tan x$, e^x and $\ln x$.		On examination papers: students will not be required to prove these results.
	Derivative interpreted as a gradient function and as rate of change.	Finding equations of tangents and normals. Identifying increasing and decreasing functions.	
	Derivatives of reciprocal circular functions. Derivatives of a^x and $\log_a x$. Derivatives of $\arcsin x$, $\arccos x$, $\arctan x$.		**See SL guide**

Topic 7—Core: Calculus (continued)

	Content	Amplifications/inclusions	Exclusions
7.2	Differentiation of a sum and a real multiple of the functions in 7.1.		
	The chain rule for composite functions. Application of chain rule to related rates of change.		
	The product and quotient rules.		
	The second derivative.	Familiarity with both forms of notation, $\frac{d^2 y}{dx^2}$ and $f''(x)$, for the second derivative.	
	Awareness of higher derivatives.	Familiarity with the notations $\frac{d^n y}{dx^n}$, $f^{(n)}(x)$.	**See SL guide**
7.3	Local maximum and minimum points.	Testing for the maximum or minimum using change of sign of the first derivative and using sign of second derivative.	
	Use of the first and second derivative in optimization problems.	Examples of applications: profit, area, volume.	

Topic 7—Core: Calculus (continued)

	Content	Amplifications/inclusions	Exclusions
7.4	Indefinite integration as anti-differentiation.	Indefinite integral interpreted as a family of curves.	
	Indefinite integral of x^n $(n \neq -1)$, $\sin x$, $\cos x$, e^x, $\frac{1}{x}$.	$\int \frac{1}{x} dx = \ln\lvert x\rvert + C$.	
	The composites of any of these with the linear function $ax + b$.	Examples: $f'(x) = \cos(2x+3) \Rightarrow f(x) = \frac{1}{2}\sin(2x+3) + C$.	**See SL guide**
7.5	Anti-differentiation with a boundary condition to determine the constant term.	Example: if $\frac{dy}{dx} = 3x^2 + x$ and $y = 10$ when $x = 0$, then $y = x^3 + \frac{1}{2}x^2 + 10$.	
	Definite integrals.		
	Area between a curve and the x-axis or y-axis in a given interval, areas between curves.	$\int_a^b y dx$ and $\int_a^b x dy$.	
	Volumes of revolution.	Revolution about the x-axis or the y-axis. $V = \int_a^b \pi y^2 dx$, $V = \int_a^b \pi x^2 dy$.	**See SL guide**

Topic 7—Core: Calculus (continued)

	Content	Amplifications/inclusions	Exclusions
7.6	Kinematic problems involving displacement, *s*, velocity, *v*, and acceleration, *a*.	$v = \frac{\mathrm{d}s}{\mathrm{d}t}$, $a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2} = v\frac{\mathrm{d}v}{\mathrm{d}s}$. Area under velocity–time graph represents distance.	**See SL guide**
7.7	Graphical behaviour of functions: tangents and normals, behaviour for large $\lvert x \rvert$; asymptotes.	Included: both “global” and “local” behaviour. Oblique asymptotes.	
	The significance of the second derivative; distinction between maximum and minimum points.	Use of the terms “concave up” for $f''(x) > 0$, “concave down” for $f''(x) < 0$.	
	Points of inflexion with zero and non-zero gradients.	At a point of inflexion $f''(x) = 0$ and $f''(x)$ changes sign (concavity change). $f''(x) = 0$ is not a sufficient condition for a point of inflexion: for example, $y = x^4$ at $(0,0)$.	Points of inflexion where $f''(x)$ is not defined, for example, $y = x^{1/3}$ at $(0,0)$. **See SL guide**
7.8	Implicit differentiation.		

Topic 7—Core: Calculus (continued)

	Content	Amplifications/inclusions	Exclusions
7.9	Further integration: integration by substitution integration by parts.	Limit changes in definite integrals. On examination papers: unusual substitutions may be given. Examples: $\int x \sin x \, \mathrm{d}x$ and $\int \ln x \, \mathrm{d}x$. Repeated integration by parts: examples: $\int x^2 \mathrm{e}^x \, \mathrm{d}x$ and $\int \mathrm{e}^x \sin x \, \mathrm{d}x$.	Integration using partial fractions. Reduction formulae.
7.10	Solution of first order differential equations by separation of variables.		